Modeling of Wireless Power Transfer Cell With Planar Circular Spiral Structure

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Wireless power transfer (WPT) technique based on magnetic resonant coupling has become a topical issue for academicians as well as engineers since it was reported by a research group of Massachusetts Institute of Technology (MIT). In this paper, a model based on partial element equivalent circuit (PEEC) is built to calculate the characteristics of WPT cells with planar circular spiral structure. The track of one turn in WPT coil is modeled as a complete ring carrying constant current, and the mutual inductances and the parasitic capacitances among different turns are considered. The self-inductances and mutual inductances between various rings are calculated by Neumann's integral. Then, the resonant frequency and scattering parameter (S_{21}) of WPT cells are calculated and measured, and the results show the modeling method has a better accuracy. Furthermore, the magnetic fields distributions of WPT cells at resonant and nonresonant frequency are calculated to show the difference between them.

***Index Terms*—Partial element equivalent circuit (PEEC), resonant frequency, scattering parameter, wireless power transfer (WPT).**

I. Introduction

Wireless power transfer (WPT) via magnetic resonant coupling (MRC) was proposed by a research team of Massachusetts Institute of Technology (MIT) in 2007 [1]. The WPT system based on MRC (MRC-WPT) consists of primary power transmitting coil and secondary power receiving coil, and power transmission is implemented when the primary coil and secondary coil resonate simultaneously. Therefore, how to design the WPT coil and emulate its characteristic of impedance, resonant frequency and power transfer efficiency are important for theory and practice.

Until now, MRC-WPT system has been analyzed by using coupled-mode theory (CMT) [2] and lumped circuit theory [3]. However, it is inconvenient to design MRC-WPT cell by CMT, and difficult to get precise lumped parameters. MRC-WPT system is operated in a frequency around a dozen MHz, so the effect of parasitic parameters cannot be ignored. The performance indexes are also decided by the structure of MRC-WPT cell in this frequency band. In paper [4], an equivalent distributive network of a coil at high frequency was built, and the distribution network parameters were calculated. But the performance indexes of the coil were extracted by the associated lumped circuit model. In paper [5] and paper [6], WPT cells with rectangle spiral structure were modeled and analyzed by using the method of partial element equivalent circuit (PEEC), but calculation of scattering parameter was not involved.

In this paper, practical MRC-WPT coils with planar circular spiral and direct feeding structure are manufactured and the coils' impedance characteristic and scattering parameters are measured. Considering the effect of parasitic parameters in high frequency band, the calculation model is built based on the PEEC method. The partial self- and mutual inductances are calculated using Neumann's formula. What's more the characteristic parameters, such as the resonant frequency and scattering parameters, are calculated based on PEEC model, and the results are in good agreement with the measured results. According to the results of PEEC, the magnetic fields distributions of WPT cells working on resonant and nonresonant modes are calculated, and the results show the magnetic fields distributions are different at the two modes.

II. Methodology

A. PEEC modeling

A WPT coil with planar circular spiral structure is described in Fig. 1 (a). In order to implement PEEC modeling, the whole WPT coil is segmented several parts as in Fig. 1 (b) . The WPT coil is split by turns. The track of one turn is treated as a complete ring carrying constant current [7]. Thus each turn of WPT coil is modeled by its self-inductance and loss resistance. The mutual inductances and parasitic capacitances between different turns, no matter adjacent or non-adjacent, are considered as well, although they are not all marked in Fig. 1 (b).

(a) (b)

Fig. 1. (a) Geometry of MRC-WPT coil. (b) Segmenting circular spiral WPT coil based on PEEC method.

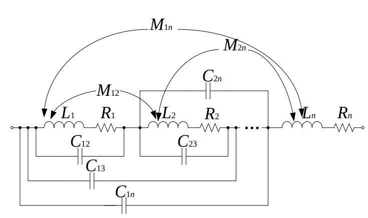


Fig. 2 PEEC network.

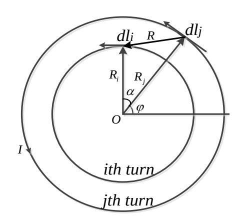


Fig. 3 Two optional rings carrying constant current.

According to above-mentioned method, the PEEC network of a WPT coil is shown in Fig. 2. There are only 3 turns for diagram simplicity. In the PEEC network used in this paper, all of the distributed parameters are considered, and only the parasitic capacitances are moved forth for half an element for calculation simplicity. The PEEC network of WPT transceiver unit is coveniently obtained by using Fig. 2.

B. Parameters calculation

Matrices of self- and mutual inductances, parasitic capacitances, and resistances of the PEEC network are derived for obtaining characteristic parameters of the WPT coil.

To calculate self- and mutual inductances, two optional rings of the coil in Fig. 1 (b) are used to explain the calculation process, and shown in Fig. 3. It is assumed that the radius of the *i*th turn is R_i , and the radius of *j*th turn is R_j.

The self-inductances and turn-to-turn mutual inductances can be calculated by using Neumann's formula as

$$L_{ji} = \frac{\mu_0}{4\pi} \oint_{l_j} \oint_{l_i} \frac{\mathrm{d}\vec{l}_i \bullet \mathrm{d}\vec{l}_j}{R} \tag{1}$$

where L_{ji} is the mutual inductance between turn i and turn j, R is given by $R = \sqrt{R_i^2 + R_j^2 - 2R_iR_j\cos\alpha}$. Therefore,

$$L_{ji} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{R_iR_j\cos\alpha \mathrm{d}\alpha \mathrm{d}\varphi}{\sqrt{R_i^2 + R_j^2 - 2R_iR_j\cos\alpha}} \tag{2}$$

where μ_0 is the magnetic constant. In (2), if j equals to i, the self-inductance of *i*th turn is obtained.

The turn-to-turn parasitic capacitances are calculated by [8],

$$C_{ij} = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \varepsilon_0 \frac{2\pi R_i r\cos\theta}{d + 2r(1-\cos\theta)} \mathrm{d}\theta \tag{3}$$

where r is the cross-section radius of the WPT coil, ε_0 is the permittivity of air, d is the separation between turns, and θ is the effective cross-section angle.

Considering the skin and proximity effects, the resistance of a circular coil can be expressed as [4]

$$R_{ac}(f) = R_{dc}\left[1 + (f/f_h)^2\right] \tag{4}$$

where R_{ac} is ac resistance of the coil, R_{dc} is dc resistance of the coil, and f is the operating frequency. The f_h is the frequency at which the ac power dissipation is twice the dc power dissipation.

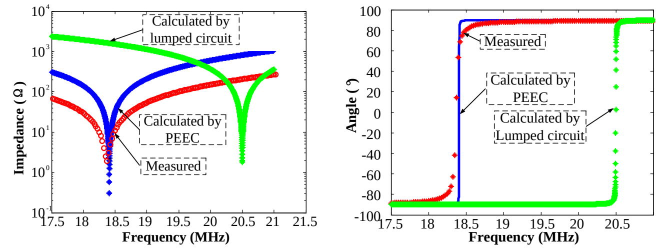
III. Results

In order to verify the accuracy of the calculation results based on PEEC modeling, a pair of WPT cells are designed, and the geometric parameters are described in Table I.

TABLE I
Geometric Parameters of WPT Cell

Parameters	Values
Inner radius of the coil	135mm
Radius of wire	0.5mm
Turns	5
Pitch	5mm

Excitation is fed to the center of WPT coil. Agilent 4395A vector network analyzer (VNA) is used to measure impedance and scattering parameters. The impedance-frequency characteristic and phase-frequency characteristic are shown separately in Fig. 4 (a) and (b), including the calculated results based on PEEC modeling, calculated results based on lumped circuit method proposed in paper [4], and measured results. It can be observed that the PEEC model is more fit with the measurement than the lumped circuit model. The resonant frequency can be obtained from the curve, which can guide the design of WPT coil.



(a) (b)

Fig. 4. (a) Impedance-frequency characteristic of WPT coil. (b) Phase-frequency characteristic of WPT coil.

The scattering parameters S_{21} of WPT cells are measured at the resonant frequency of 18.41MHz as the distance between sending and receiving WPT coils is varied from 10 to 65 cm in steps of 5 cm. The calculated results based on PEEC modeling and measured results of S_{21} are shown in Fig. 5. It can be noted that the shapes of two curves coincide well.

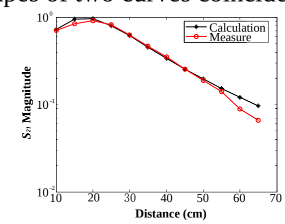


Fig. 5. Variations of the S_{21} with distances of sending and receiving WPT coils at the resonant frequency of the coil.

IV. Conclusion

In this paper, PEEC method is used to model the WPT cells and calculate characteristic parameters for MRC-WPT cells. The impedance-frequency characteristic, phase-frequency characteristic and scattering parameter (S_{21}) of MRC-WPT cells are calculated and evaluated by comparing the results with those obtained experimentally. It is found that the PEEC method has higher precision than that of the lumped circuit method. Using this method, it is convenient and efficient in obtaining characteristics of the cells at design stage.

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